Problem 1 (6c, 8c):

Use Lagrange interpolating polynomials to approximate if , , , .

In order to find the first- and second-degree interpolating polynomials, use the first two and the first three points respectively. The full interpolation will produce a third-degree polynomial. Using Lagrange interpolation in MatLab gives the following interpolating polynomials:

(1)

(2)

(3)

Evaluating these at gives:

(4)

(5)

(6)

The function was generated by . Use the error formula to find the bound on the error and compare it to the actual error for .

The error for Lagrange Polynomials is given by:

(7)

First begin by finding the higher derivatives of necessary for the error calculations. For and , is maximized at and respectively. This gives:

(8)

(9)

Using MatLab to solve for where is maximized on the considered interval (by finding where the derivative of ) gives the following upper bounds on error for and .

(10)

(11)

The actual error at these points is given by:

(12)

(13)

Problem 2 (20):

Use Lagrange interpolation to approximate the average weight curve for each sample and find the approximate maximum average weight for each sample via the maximum of the interpolating polynomial.

Following the definition in the previous problem, the two interpolating polynomials and (which corresponds to sample 1 and sample 2) are given by:

(1)

(2)

Finding the x-values on the interval where these polynomials have derivatives equal to zero give the possible location of maxima, and plugging in these possible values allow the determination of the maximum average weight for each sample. For these polynomials, this gives:

(3)

(4)

Problem 3 (2c):

Use Neville’s method to obtain the approximations for Lagrange interpolating polynomials of degrees one, two, and three for the function described in problem 1.

Using the same points in problem 1 to give each approximation gives:

(1)

(2)

(3)

Note that the first column contains the x-values used, the second column contains the function at those x-values, and the lower right entry is the final approximation. The approximations are the same as those obtained in problem 1.

(4)

(5)

(6)

Problem 4 (10):

Consider an incorrect approximation using Neville’s Method of using where the correct values are and compare the error.

To accomplish this, define all of the original functions as , which can be represented by the vector:

(1)

The correct vector can thus be represented by:

(2)

Running both of these through Neville’s Method gives the following approximations for :

(3)

(4)  
The incorrect values of produce an approximation that is less than the correct approximation for .

Problem 5 (18):

Find the interpolating polynomial to predict a ¾ mile time using divided differences.

The divided differences formula gives the following coefficient matrix:

(1)

Where the coefficients are contained on the main diagonal. Labeling the coefficients starting with one on the top left and incrementing by one moving down the diagonal, the polynomial is given by:

(2)

For this problem, this gives:

(3)

Taking gives:

(4)

This gives an absolute error of .

Problem 6 (9):

Use Hermite interpolation to interpolate the function.

The resulting coefficient matrix is:

(1)

This gives the following polynomial:

(2)

Which, at , evaluates to:

(3)

Problem 7 (25):